CRACK-LIKE AND PULSE-LIKE DYNAMIC FRICTIONAL SLIDING

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ABSTRACT

Numerical and experimental investigation of frictional sliding under dynamic loading conditions is discussed. The configuration analyzed consists of two plates of Homalite (an elastic birefringent polymer material) connected along a planar interface. The plates are characterized as isotropic elastic materials and the interface is characterized by a rate- and state-dependent frictional law that also accounts for dependence on normal stress variations. The calculations are carried out within a framework where two constitutive relations are used: a volumetric constitutive relation between stress and strain and a surface constitutive relation that characterizes the frictional behavior of an interface. The propagation speeds of the sliding tip are found to be of the order of the longitudinal wave speed. Frictional sliding is found to occur in modes that involve uniform sliding behind the rupture front, an isolated slip pulse, multiple slip pulses or a combination of these modes. The dependence of the sliding mode on the initial compressive stress, the impact velocity and the friction parameters is described. Numerical results compare favorably with experimental observations in terms of intersonic sliding tip speed, crack-like and pulse-like sliding modes and the stress fields at the sliding tip.

1 INTRODUCTION

Frictional sliding along an interface between two deformable solids is a basic problem of mechanics that arise in a variety of contexts including, for example, material processing, deformation and failure of fi ber reinforced composites and earthquake dynamics. The classical Coulomb type of frictional relation relates the shear stress to the normal stress by a proportionaliy constant μ which can have a dependence on the relative sliding velocity. However, Adams (1995) and Ranjith and Rice (2001) showed that the problem of frictional sliding along an interface between two elastic solids, with sliding governed by Coulomb friction, is unstable to perturbations and hence ill-posed for a signifi cant range of values of μ. Rate- and state-dependent models of friction have been introduced (Dieterich, 1979; Rice and Ruina, 1983; Ruina, 1983) that phenomenologically characterize the surface evolution and provide a representation of the transition from static to dynamic friction at constant normal load. For varying normal stress, Prakash and Clifton (1993) added an additional state variable to account for their observation of a delay in the change in shear traction following a sudden change in the normal traction. The use of these friction laws regularizes the sliding friction problem of elastic bodies with changing normal stress and make it a well-posed problem. We report on the implementation of a rate-state friction model in a fi nite-element code to simulate the frictional sliding behavior of two elastic solids under dynamic loading conditions. The computational results of dynamic sliding are compared with experimental observations.

2 FORMULATION AND NUMERICS

In a fi nite strain Lagrangian formulation, with the initial undeformed confi guration taken as reference and with all fi eld quantities considered to be functions of convected coordinates y^i and time t, the principle of virtual work can be written as

$$\int_V \mathbf{S} : \delta\mathbf{E} dV - \int_{S_{int}} \mathbf{T} \cdot \delta[\mathbf{u}] dS = \int_{S_{ext}} \mathbf{T} \cdot \delta\mathbf{u} dS - \int_V \rho \frac{\partial^2 \mathbf{u}}{\partial t^2} \cdot \delta\mathbf{u} dV \tag{1}$$

where $\mathbf{S}$ is the second Piola-Kirchhoff stress tensor, $\mathbf{u}$ is the displacement vector, $[\mathbf{u}]$ is the displacement jump across the cohesive surface, $\mathbf{A} : \mathbf{B}$ denotes $A^{ij}B_{ji}$, and V, S_{ext} and S_{int} are the volume, external surface area and internal cohesive or frictional surface area, respectively, of the body in the reference confi guration. The density of the material in the reference confi guration is ρ, $\mathbf{T}$ is the traction vector and the Lagrangian strain, $\mathbf{E}$, is given by

$$\mathbf{E} = \frac{1}{2}(\mathbf{F}^T \cdot \mathbf{F} - \mathbf{I}) \quad , \quad \mathbf{F} = \mathbf{I} + \frac{\partial \mathbf{u}}{\partial \mathbf{x}} \tag{2}$$

with $\mathbf{I}$ the identity tensor and $\mathbf{x}$ denoting the position vector of a material point in the reference confi guration.

A confi guration studied is sketched in Fig. 1. At $t = 0$, the body is at rest, an initial compressive stress is prescribed and impact loading is imposed. The calculations are two dimensional assuming plane stress conditions. A frictional constitutive relation is prescribed along the centerline and the material is taken to be linear elastic.

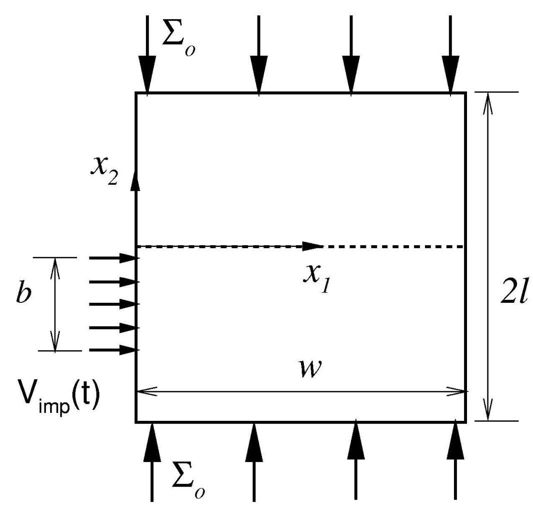

Figure 1: Geometry and loading confi guration used in the fi nite element calculation.

The friction constitutive law used is a modifi ed version of the Prakash-Clifton rate- and state-dependent law (Coker, et al. 2004) characterized by the relation,

$$T_s = \mu(\theta_0, V_{slip})(\theta_1 + \theta_2) \tag{3}$$

where V_{slip} is the relative sliding velocity of the interface, T_s is the shear traction at the interface, θ_i are the state variables characterizing the history of the interface. The evolution equations for the state variables also include a characteristic length and the parameters are chosen so that the shear traction decreases with sliding velocity (velocity-weakening) as shown in Fig. 2a. The behavior in the apparent coefficient of friction, T_s/T_n that this law exhibits when subjected to a step jump in sliding velocity is shown in Fig. 2b.

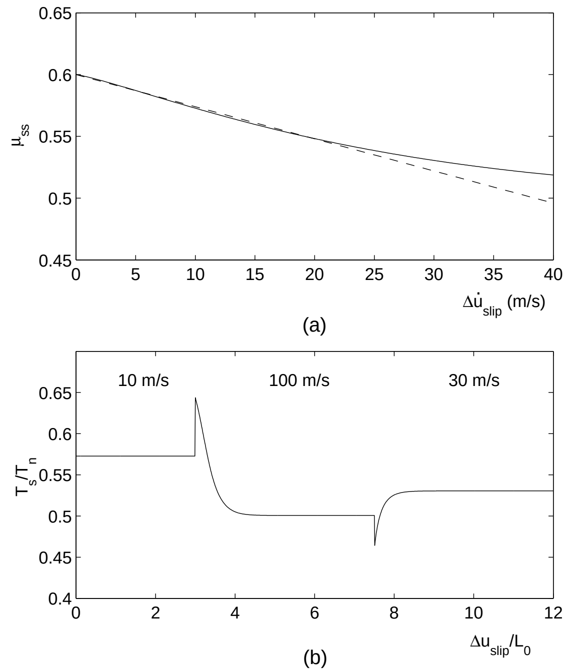

Figure 2: (a) Steady-state coefficient of friction, μ_{ss}, as a function of the sliding rate, $\Delta\dot{u}_{slip}$. (b) The effect of an abrupt change in the sliding rate on the apparent coefficient of friction for the rate- and state-dependent friction relation used in the calculations.

A finite element discretization is used that is based on linear displacement triangular elements that are arranged in a 'crossed-triangle' quadrilateral pattern. The equations of motion that result from substituting the finite element discretization into (1) are integrated numerically by an explicit integration procedure, the Newmark β-method with $\beta = 0$, Belytschko et al. (1976). A lumped mass matrix is used instead of the consistent mass matrix, since this has been found preferable for explicit time integration procedures, from the point of view of accuracy as well as computational efficiency, Krieg and Key (1973).

The experimental procedures used for the configuration sketched in Fig. 1 are similar to those used to study shear crack propagation (Rosakis et al., 1999). The impact loading is imposed via a cylindrical steel projectile fired using a gas gun with impact speeds ranging from 10 m/s to 60 m/s. Dynamic photoelasticity is used to extract stress field information around the interface. The photoelastic fringe patterns (isochromatic fringes that are the contours of the difference of maximum and minimum in-plane principal stresses) were recorded in real time using a high-speed Cordin CCD camera capable of capturing 16 images at a rate of 100 million frames per second.

3 RESULTS

Finite element calculations using rate- and state-dependent friction laws for the interface showed that sliding of an interface can occur in different modes depending on the imposed compressive stress and the impact conditions. Friction modes that involve uniform sliding behind the rupture front, an isolated slip pulse, multiple slip pulses or a combination of these were obtained. It is found that the isolated slip pulse tends to become narrower and steeper with time while the multiple pulses do not. The experiments also exhibited both pulse-like and crack-like modes of sliding with a transition from crack-like mode to pulse-like mode with increasing impact speeds. In both experiments and calculations Mach lines in the photoelastic stress contours are observed. Calculations from the angle of the Mach lines show that the sliding tip travels at a speed greater than $\sqrt{2}c_s$ and that a trailing pulse travels faster than c_l. The results are presented in Coker et al. (2004).

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